Section 1.2:
Cartesian product

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Example:

$$\mathbb{R} \times \mathbb{R} = \mathbb{R}^2 = \{(x, y) : x \in \mathbb{R}, y \in \mathbb{R}\}$$

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Example: Let $A = \{1, \varnothing\}$ and $B = \{1, b, \{\rho\}\}$, then

$$A \times B = \{(1,1), (1,b), (1,\{\rho\}), (\varnothing,1), (\varnothing,b), (\varnothing,\{\rho\})\}$$
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Theorem

If A and B are finite sets, then $|A \times B| = |A| \cdot |B|$.

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Let $A_1, A_2, \ldots, A_n$ be sets. Then

$$A_1 \times A_2 \times \cdots \times A_n = \{(a_1, a_2, \ldots, a_n) : a_i \in A_i \text{ for } 1 \leq i \leq n\}.$$

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Example: Let $A = \{1\}$, $B = \{\alpha, \beta\}$, and $C = \{\circ, \square\}$, then

$$A \times B \times C = \{(1, \alpha, \circ), (1, \alpha, \square), (1, \beta, \circ), (1, \beta, \square)\}$$
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Definition

If A is a set, then $A^n = \underbrace{A \times \cdots \times A}_{n}$.

Homework.

1. Read Section 1.2.
2. Write up the following exercises.
   Section 1.2: 2.e, 2.f, 5, 8.

New LaTeX commands

$A \times B$	`A \times B`
$A^{superscript}_{subscript}$	`A^{superscript}_{subscript}`